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"A NEW APPROACH TO CALCULATE SINUSOIDAL VALUES"

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ABSTRACT

Sinusoidal functions are one of the most fundamental functions for STEM branch. Every student from his secondary standard and onwards experiences their necessity at every next step. Motive of this article is to enlighten schooling students about an equation which provides satisfactory sine values. This method is based upon the linear interpolation for an interval of 5º. Obtained values are very much close to their natural values and hence, are helpful for rough estimation of engineering problems in the absence of trigonometric tables and calculator. Methodology applied to obtain this equation is purely analytical and have no any derivational roots. Maximum deviation of obtained values from their sinusoidal values is 0.009 only so it can be applied widely if change in 1º hardly matters.

KEYWORDS: Arithmetic Progression (A.P), Greatest Integer Function [X], Linear Interpolation Reminder Operator (a % b)

INTRODUCTION

Structure of today's trigonometry is based on pillars of work of many mathematicians belonging to different continents, distinct civilizations and for a wide time span. It dates back to the early ages of Egypt and Babylon about 1500 BC. Trigonometry was then advanced by the Greek astronomer Hipparchus who put a trigonometry table that measured the length of the chord subtending the various angles in a circle of a fixed radius r. His work was taken further by Ptolemy by creating the table of chords with increment of 1° which was known as Menelaus's theorem. Angle measurement in degree was started by Babylonians and we are using it till now. Around the same period, Indian mathematicians created the new trigonometry system based on the sine function instead of the chords. Trigonometry also includes appreciable contribution of Muslim astronomers who compiled both the studies of the Greeks and Indians in the middle age. From13th century and onwards, modern trigonometry was enhanced by Europeans like Isaac Newton, Euler by defining trigonometry functions as ratios rather than lengths of lines.

The application of trigonometry came about primarily due to the purposes of astronomy, surveying, navigation, construction and now it has extended so far [1]. Siddhantas and Aryabhatia (work of Indian Mathematicians) contain the earliest surveying table of sine values (jyā table) in 3°45' from 0° to 90°, to an accuracy of 4 decimal places [2]. Investigations were made on circle of radius 3438 units. Later on Bhaskara in 7th century provided a formula to obtain sine values with a relative error less than 1.8% [3] [4]. First accurate sine table was produced by Muhammad ibn Mûsâ al Khwârîzmî in early 9th century.

Purpose

We all are very much familiar with multidisciplinary nature of trigonometry. It plays a vital role in solving diverse problems of STEM field. Many times, we are known to the complete procedure to find out solution but problem get left due to unavailability of trigonometric values. There are several tactics to obtain sine value of an angle very accurately with high precision like Tayler series, Bhaskara II formula and many other trigonometric identities but it become cumbersome to get sum of first 3 or 4 terms of Madhava's sine series. Mugging up these many formulae may also lead to confusion and tedious to deduce values from them. Equation provided in article, obtained analytically, provides very good approximation. It gives sinusoidal value of angles which are very close to their natural values. Once, the value of sin θ is found other trigonometric ratios can also be found. I hope that it would be fun learning mathematics for secondary and senior secondary students

Method Applied

Purely analytical method is adopted for the development of formula. On considering the difference between two consecutive sine values for angles being multiple of 5, a pattern is obtained which has consecutive differences in which sine values are in pair-wise A.P amazingly! But a small deviation occurs at beginning and at end.

From the $\theta = 15$, a pair wise A.P get formed,

Sum of first two terms which are not in A.P is $16*10^{-2} = 0.16$

In this pair wise AP 9, 9 is first term, 8, 8 is second term and so on. As 9, 9 is first term occurs for $\theta = 15$ and $\theta = 20$.

So, Number of term (n) can be given by-

$n = [(\theta-5)/10]$

Where, [X] is Greatest integer value which is smaller or equal X

Nth term a_n of an A.P. is given as

$$a_n = a_1 + (n-1)*d \tag{1}$$

And, sum of first n terms (S_n) of an AP is given as

$$S_n = n/2 *[2a_1 + (n-1)*d] \tag{2}$$

Where, n: number of term, a_1: First term, d: Common difference

For the obtained series we have

$a_1 = 0.09$ and $d = -0.01$

On putting values of a_1 and d in equation (1) and (2) we get,

$$a_n = a_1 + (n-1)*d$$

$$a_n = 0.09 + (n-1)*(-0.01)$$

$$a_n = \{10-n\}*10^{-2}$$

$$a_n = \{10 - [(\theta-5)/10]\}*10^{-2}$$

$$S_n = n/2 *\{2a_1 + (n-1)*d\}$$

$$S_n = n/2*\{2*0.09 + (n-1)*(-0.01)\}$$

$S_n = n/2*\{18-n+1\}*10^{-2}$

$S_n = n/2*[19-[(\theta-5)/10]]*10^{-2}$

Addition of Correction Terms

The above mentioned equation for S_n could provide very accurate values for angles in between 15° to 70°. So, two correction terms, one of the terms corrects sinusoidal values till 10° and another term corrects values from 75° to 90°, are made added to make it applicable for all angles. For $\theta \leq 10°$, term $[2^{[(10-\theta)/5]}]*10^{-2}$ is added which provides required corrective values for $\theta \leq 10°$ and 0 for other angles. And for $75° \leq \theta \leq 90°$, term $[(\theta-70)/5]*[\theta/70]*10^{-2}$ is added which provides required corrective values for $75° \leq \theta \leq 90°$ and 0 for other angles. Here term $[(\theta-70)/5]$ provides the corrective value and term $[\theta/70]$ restrict it to work only for $\theta \geq 70°$.

So equation becomes,

$Y(\theta) = \{0.16 + S_n + \text{correction terms}\}$ //0.16 is sum of terms not in A.P

$Y(\theta) = \{16 + [(\theta-5)/10] * (19-[(\theta-5)/10]) - (\theta \%2)*(10 - [\theta/10])$

$+ [2^{[(10-\theta)/5]}] - [(\theta-70)/5]* [\theta/70]\} *10^{-2}$

Where, % is reminder operator and θ is multiple of 5.

Generalization to All Angles

From our previous exercises, we are familiar with the obtained pattern. Difference in values occurs in pair wise A.P for an interval of 5° angle. As for every interval, difference varies in A.P, so we can easily apply linear interpolation to deduce values in between them.

For every value of θ, breaking it in two components as

$\theta = 5p + q$

Where, p is a positive integer and $0 \leq q \leq 4$

Let, $Y(\theta) = Y(5p) + D(\theta)$

Where, $D(\theta) = q*(Y(5p+5)-Y(5p))/5$

So, $Y(\theta) = (1-q/5)\,Y(5p) + q*(Y(5p+5))/5$

RESULTS

Values obtained by equations are very close to sine values with high accuracy. Approximate sine values can be deduced for every angle. One example is solved below:

Obtained equation is as,

$Y(\theta) = (1-q/5)*Y(5p) + q*(Y(5p+5))/5$ (1) For any general value of θ

Where $5p \leq \theta < 5p+5$

And,

Y (θ) = {16 + [(θ-5)/10] * (19-[(θ-5)/10]) – (θ %2)*(10 - [θ /10])

+ [$2^{[(10-θ)/5]}$] – [(θ-70)/ 5] *[θ/70]}*10^{-2} (2) For θ multiple of 5

Table 1

Θ	[(θ -5)/10]	[2[(10-θ)/5]]	[(θ -70)/5] *[θ /70]	[θ /10]	(θ %2)	Y(θ)	Sin θ	Deviation = Sin θ - Y(θ)
0	-1	4	0	10	0	0.00	0.00	0
5	0	2	0	10	1	0.08	0.087155	0.007156
10	0	1	0	9	0	0.17	0.173648	0.003648
15	1	1	0	9	1	0.25	0.258819	0.008819
20	1	0	0	8	0	0.34	0.342020	0.002020
25	2	0	0	8	1	0.42	0.422618	0.002618
30	2	0	0	7	0	0.50	0.5	0
35	3	0	0	7	1	0.57	0.573576	0.003576
40	3	0	0	6	0	0.64	0.642787	0.002787
45	4	0	0	6	1	0.70	0.707106	0.007106
50	4	0	0	5	0	0.76	0.766044	0.006044
55	5	0	0	5	1	0.81	0.819152	0.009152
60	5	0	0	4	0	0.86	0.866025	0.006025
65	6	0	0	4	1	0.90	0.906307	0.006307
70	6	0	0	3	0	0.94	0.939692	-0.003073
75	7	0	1	3	1	0.96	0.965925	0.005925
80	7	0	2	2	0	0.98	0.984807	0.004807
85	8	0	3	2	1	0.99	0.996194	0.006194
90	8	0	4	1	0	1.00	1.00	0

Values of every individual term of expression Y (θ) is mentioned in various columns. Obtained values Y (θ) and sine values are very close to each other. Y (θ) is obtained by the equation

Y (θ) = {16+ [(θ-5)/10] * (19-[(θ-5)/10]) – (θ %2)*(10 - [θ /10])

+ [$2^{[(10-θ)/5]}$] – [(θ-70)/ 5]* [θ/70]} *10^{-2}

Most of the time accuracy is more than 99%.

GRAPHS

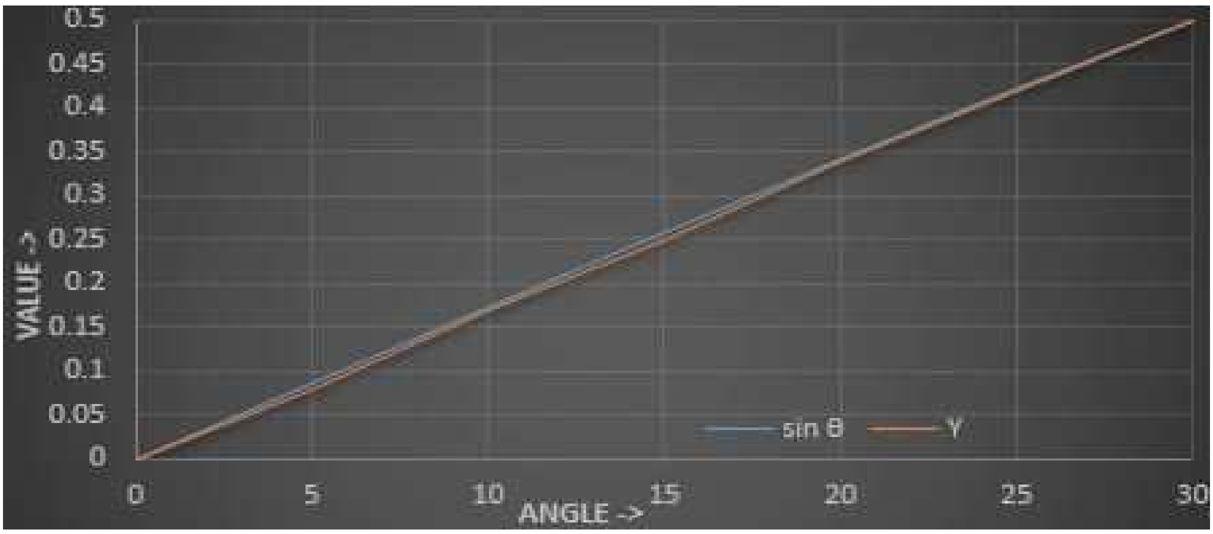

Figure 1

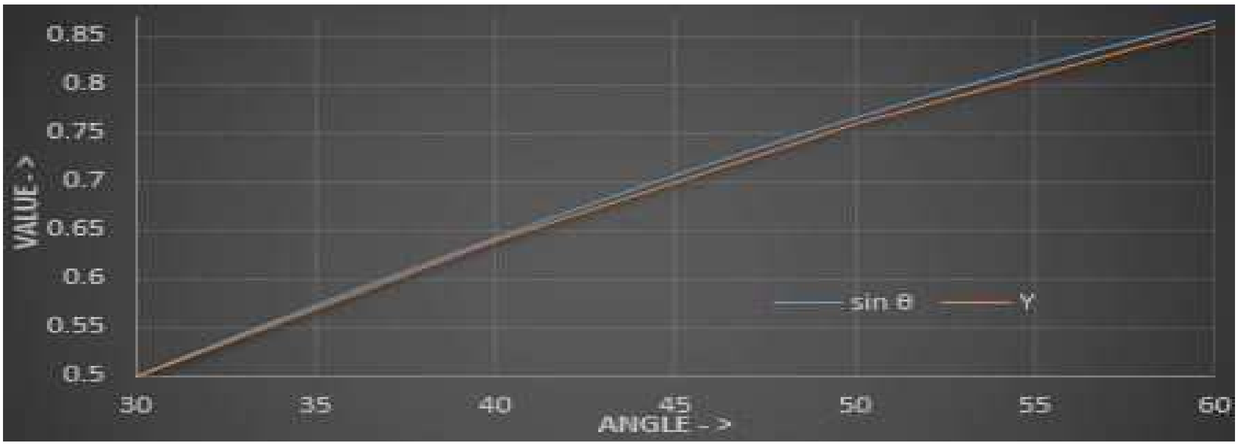

Figure 2

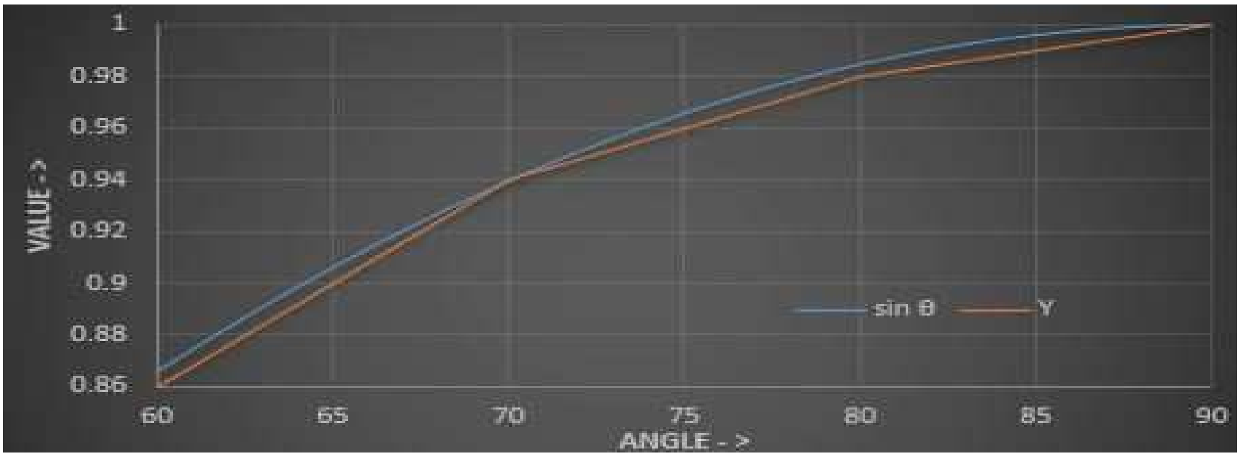

Figure 3

Graphs are potted between angle (on X – axis in degree) and their corresponding 'Natural sine value' and 'obtained value' (on Y – axis). Natural sine values are indicated by blue line and obtained values are indicated by red line. In figure 1 both of lines coincides. **Both lines are very closer to each other in every graph**.

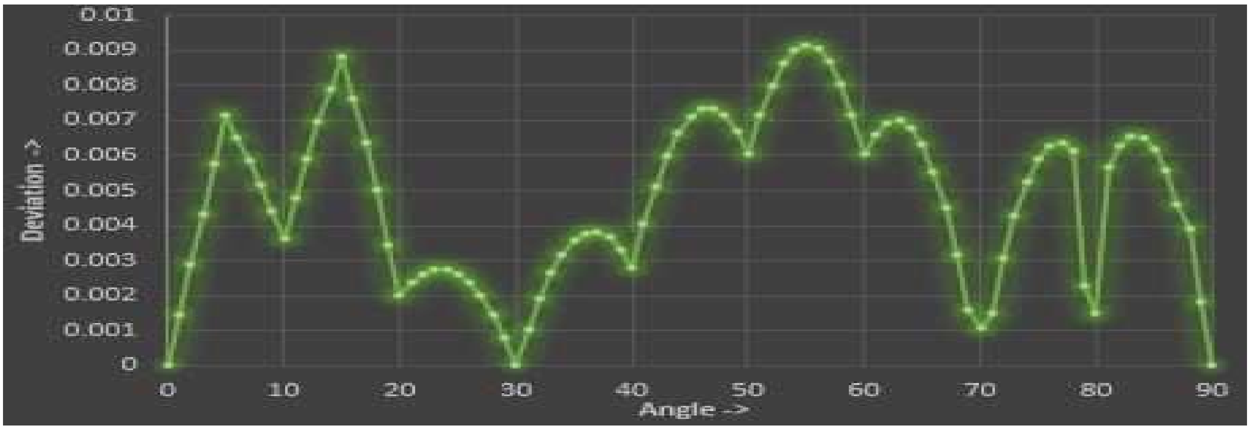

Figure 4

Graph is potted between angle (on X – axis) and corresponding difference between 'natural sine value' and 'obtained value' (on Y – axis). Deviation of obtained value from standard sine value is indicated by points. **Maximum deviation is nearly 0.009 only**. It is very close to its natural sine value 0.82903757! It is the maximum deviation given by equation.

DISCUSSIONS

Readers can do some exercises on their own, and verify the formulae by checking with the obtained values as shown in the tables and graphs. Hope that it would be very fun leaning mathematics for young school students which may encourage for innovating new mathematical riddles and looking for possible formalism if any.

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CONCLUSIONS

In this paper we derived an empirical formula which provides approximate sinusoidal values. In the absence of trigonometric table or calculator, this formula is very helpful for rough estimation of elementary science and engineering problems.

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